

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
January 2012

# Mathematics

# MFP3

## Unit Further Pure 3

Monday 23 January 2012 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J A N 1 2 M F P 3 0 1

Answer **all** questions in the spaces provided.

**1** The function  $y(x)$  satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = \frac{y - x}{y^2 + x}$$

and

$$y(1) = 2$$

**(a)** Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with  $h = 0.1$ , to obtain an approximation to  $y(1.1)$ .

(3 marks)

**(b)** Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to  $y(1.2)$ , giving your answer to three decimal places.

(3 marks)

QUESTION  
PART  
REFERENCE

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



QUESTION  
PART  
REFERENCE

Handwriting practice area with horizontal dashed lines.



0 3

Turn over ►

2

Find

$$\lim_{x \rightarrow 0} \left[ \frac{\sqrt{4+x} - 2}{x + x^2} \right]$$

(3 marks)

QUESTION  
PART  
REFERENCE



3 Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 26e^x$$

given that  $y = 5$  and  $\frac{dy}{dx} = 11$  when  $x = 0$ . Give your answer in the form  $y = f(x)$ . (10 marks)

QUESTION  
PART  
REFERENCE

A large rectangular area with horizontal dotted lines for writing the answer.





**4 (a)** By using an integrating factor, find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \ln x \qquad (7 \text{ marks})$$

**(b)** Hence, given that  $y \rightarrow 0$  as  $x \rightarrow 0$ , find the value of  $y$  when  $x = 1$ . (3 marks)

QUESTION  
PART  
REFERENCE

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





QUESTION  
PART  
REFERENCE

A large rectangular box with a vertical line on the left side. The box contains 25 horizontal dotted lines, providing a grid for writing answers.

Turn over ►



5 (a) Explain why  $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2 + 3e^{4x}} dx$  is an improper integral. *(1 mark)*

(b) By using the substitution  $u = x^2e^{-4x} + 3$ , find

$$\int \frac{x(1-2x)}{x^2 + 3e^{4x}} dx \quad (3 \text{ marks})$$

(c) Hence evaluate  $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2 + 3e^{4x}} dx$ , showing the limiting process used. *(4 marks)*

QUESTION  
PART  
REFERENCE





6 (a) Given that  $y = \ln \cos 2x$ , find  $\frac{d^4y}{dx^4}$ . (6 marks)

(b) Use Maclaurin's theorem to show that the first two non-zero terms in the expansion, in ascending powers of  $x$ , of  $\ln \cos 2x$  are  $-2x^2 - \frac{4}{3}x^4$ . (3 marks)

(c) Hence find the first two non-zero terms in the expansion, in ascending powers of  $x$ , of  $\ln \sec^2 2x$ . (2 marks)

QUESTION  
PART  
REFERENCE

Dotted lines for writing answers.









7 It is given that, for  $x \neq 0$ ,  $y$  satisfies the differential equation

$$x \frac{d^2y}{dx^2} + 2(3x + 1) \frac{dy}{dx} + 3y(3x + 2) = 18x$$

(a) Show that the substitution  $u = xy$  transforms this differential equation into

$$\frac{d^2u}{dx^2} + 6 \frac{du}{dx} + 9u = 18x \quad (4 \text{ marks})$$

(b) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} + 2(3x + 1) \frac{dy}{dx} + 3y(3x + 2) = 18x$$

giving your answer in the form  $y = f(x)$ . (8 marks)

QUESTION  
PART  
REFERENCE

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





QUESTION  
PART  
REFERENCE

Handwriting practice area consisting of 25 rows of dotted lines within a rectangular box. A vertical line on the left side separates a column for 'QUESTION PART REFERENCE' from the rest of the page.

Turn over ►



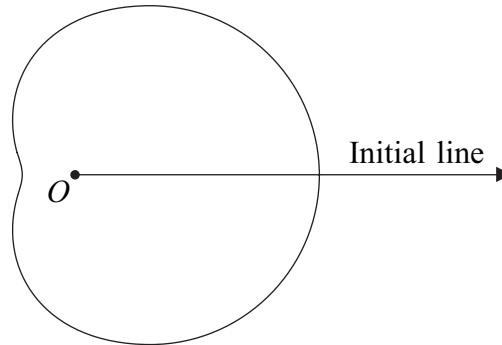




8

The diagram shows a sketch of the curve  $C$  with polar equation

$$r = 3 + 2 \cos \theta, \quad 0 \leq \theta \leq 2\pi$$



- (a) Find the area of the region bounded by the curve  $C$ . (6 marks)
- (b) A circle, whose cartesian equation is  $(x - 4)^2 + y^2 = 16$ , intersects the curve  $C$  at the points  $A$  and  $B$ .
- (i) Find, in surd form, the length of  $AB$ . (6 marks)
- (ii) Find the perimeter of the segment  $AOB$  of the circle, where  $O$  is the pole. (3 marks)

QUESTION  
PART  
REFERENCE

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



QUESTION  
PART  
REFERENCE

A large rectangular box containing a grid for writing answers. The grid has 20 columns and 30 rows. Each cell in the grid is defined by horizontal and vertical dashed lines. The top-left cell is designated for the question reference, with the text "QUESTION PART REFERENCE" printed in it.

Turn over ►





QUESTION  
PART  
REFERENCE

A large rectangular area containing horizontal dotted lines for writing, with a vertical line on the left side.

**END OF QUESTIONS**



**There are no questions printed on this page**

**DO NOT WRITE ON THIS PAGE  
ANSWER IN THE SPACES PROVIDED**

